on store—together, £250. And the loss will be divided thus: A, £75; B, £50; C, £175 = £300.

Second.—C will pay AS IF its policy insured £200 on store, and £150 on dwelling—together, £350. And the loss will be divided thus: A, £90; B, £25; C, £185 = £300.

As each of these solutions is as good as the other, it is plain that the question remains unsettled. The results, as deduced from the Summary given in this paper, would have been—C's policy applicable in the following proportions to pay the loss: on dwelling, £150, and on store, £50; together, £200, as in the policy. And the loss would have been apportioned thus: A to pay £90; B, £50; C, £160 = £300.

These plans do not exhaust all the methods of apportionment which have been tried, nor is it necessary that all should be referred to. There are different ways of testing whether any one is correct, but *in general* this may be ascertained by reference to the results. If these be tried by the 2nd Deduction, and found to lead to different amounts being insured than those which the policies admit, the erroneous nature of the premises on which the calculation rests is quite certain.

Observations on the Paper by Mr. Jellicoe, published in No. XXII. of this Journal. By* HENRY WILBRAHAM, *Esq., Fellow of Trinity College, Cambridge.*

TAKE first the case in which A has not a life interest, but one for n years, remainder to B. Let P be a sum of consols, $100-\delta$ the present price of consols; consequently, P consols sells for $P\frac{100-\delta}{100}$ cash, which may be called M; A's and B's shares of it being M_1 and M_2.

B has a right to a sum of P consols n years hence: it matters not to him how this sum arises, provided it be ready for him in n years' time. Suppose such a sum now appropriated to accumulate for n years as will produce P, B will be in the same position as if A enjoyed the sum intermediately; and if such sum be now

* "On the Valuation of Property held for Life and in Reversion; and on the due Apportionment of it, when so held on the same Life, between the Tenant for Life and the Remainder-man."

handed over to B he will neither gain nor lose. Supposing M_2 to be this sum, M_2 invested to accumulate for n years = P consols.

But how is it to be accumulated? on what stock, or at what rate of interest? I answer, that if the object be to place B in precisely the same condition as if A enjoyed the consols for n years, B has clearly a right to insist that the security for the accumulating sum shall be as good as the security he would have for the ultimate enjoyment of the consols if A have them meantime: in short, he has a right to demand that it should be invested in consols, and that the intermediate dividends should be also invested in consols. He would have a right to object to its being laid out in a 5 per cent. stock, for that high rate of interest shows that such a stock is not popularly considered so good a security as what he may justly demand. What he relinquishes now is *consol security for the possession n years hence of P consols;* that would not be compensated for by giving him instead 5 (*or* 6) *per cent. security for the possession n years hence of P consols;* it would be compensated for by consol security, though in a different form.* It is true that an Assurance Office might not give so much as would produce P in n years invested in consols; and that because (exclusive of the profit of the Company on the transaction) the Company would invest their money at higher interest, taking on themselves whatever extra risk there be in it. They, in short, are not customers who would appreciate the peculiar merits of the article B has to sell, viz., the consol security, and would only pay him for it what it would be worth if it had not this extra merit; but that does not prove that it is no merit at all, or none which ought to be regarded in dealings between private persons. If there were none, consols would be now at 70 or thereabouts instead of 94.

To pursue the investigation on this basis: $100-\delta$ being the present price of consols, $100-\delta_{\frac{1}{2}}$ the price a half year hence, $100-\delta_1$ a year hence, &c., $\frac{M\,100_2}{100-\delta}$ will be the sum of consols now bought; $\frac{100\times M_2}{100-\delta}\,\frac{1\frac{1}{2}}{100-\delta_{\frac{1}{2}}}$ the addition in consols to it made the first half year; $\frac{100\,M_2}{100-\delta}\left\{1+\frac{1\frac{1}{2}}{100-\delta_{\frac{1}{2}}}\right\}\frac{1\frac{1}{2}}{100-\delta_1}$ the addition the

* As the discussion of a subject generally serves to throw light upon it, we have inserted this paper of Mr. Wilbraham's; but it will, we think, be seen at once that the reasoning in it is altogether erroneous. Mr. Wilbraham confounds the rate of interest which the purchaser of a security, under certain limitations, may reasonably expect to make, with the rate which the security without such limitations may be yielding—the two having of course no connection. Mr. Jellicoe's paper goes to show how a surplus property, created simply by the circumstance that two proprietors concur to sell, should be disposed of: without such concurrence the surplus would have no existence.—ED. *A. M.*

second half year, and so on to the $2n$th half year; and it is easily seen that the sum of these will be

$$\frac{100\,M_2}{100-\delta}\left(1+\frac{1\frac{1}{2}}{100-\delta_{\frac{1}{2}}}\right)\left(1+\frac{1\frac{1}{2}}{100-\delta_1}\right)\ldots\ldots\ldots\left(1+\frac{1\frac{1}{2}}{100-\delta_n}\right).$$

This must $=$ P. For shortness, call $\frac{1\frac{1}{2}}{100-\delta_s}$, r_s; this being the rate of half yearly interest for the time being derivable from an investment in consols.

$$M_2=\frac{100-\delta}{100}\cdot\frac{P}{(1+r_{\frac{1}{2}})(1+r_1)\ldots\ldots(1+r_n)}.$$

If we could consider the price of consols invariable, and therefore $1+r_{\frac{1}{2}}=1+r_1=\ldots\ldots=1+\frac{1\frac{1}{2}}{100-\delta}$, or $1+r$, this would equal $\frac{100-\delta}{100}\frac{P}{(1+r)^{2n}}$; or if we could consider the present price of consols to be the average price—or rather, if we could suppose $1+r$ to be the geometric mean of all values during n years, which would give $100-\delta$ somewhat *greater* than the true average price, since the arithmetic mean of any number of quantities is always greater than the geometric mean—this last value would also be correct.

If however the present price be either considerably higher or lower than a mean price, M_2 will by such a formula be given greater or less than it ought to be. Suppose $1+r^1$ or $1+\frac{1\frac{1}{2}}{100-\delta^1}$ to be the geometric mean of the successive values of $1+r$ during a sufficiently long succession of years—suppose 20 or 30 years—from past experience, which is found by taking the arithmetic mean of the values of log. $(1+r)$, the true value of M_2 should be between $\frac{100-\delta}{100}\cdot\frac{P}{(1+r)^{2n}}$ and $\frac{100-\delta}{100}\frac{P}{(1+r^1)^{2n}}$, approximating nearer to the *former* or *latter* limit as the time n years is *less* or *greater*.

I assume here that there really is a mean price of consols, about which the actual price fluctuates; which mean price is the same for one series of 20 or 30 years and another, and which may therefore be arrived at from past experience; and, consequently, that any actual difference between the present and mean price is transitory. The difficulty—an insuperable one, so far as absolute accuracy is required—is to say, how long and to what extent this transitory state may be expected to last. Suppose we assume that in k years it may be expected that, so far as the causes which pro-

duce the present peculiarly high or low price are concerned, consols will have returned to their mean price; then,

$$M_2 = \frac{100}{100-\delta} \frac{P}{(1+r_{\frac{1}{2}})(1+r_1)\ldots(1+r_k)(1+r^1)^{2n-2k}};$$

and if during the next k years we assume that the mean price will be the mean between the present price $100-\delta$ and $100-\delta^1$,

$$M_2 = \frac{100}{100-\delta} \frac{P}{(1+r)^{2k}(1+r^1)^{2n-2k}}.$$

Again, take the case as regarded from A's point of view. A will be in the same condition as at present, if a sum M_1 be invested in consols, and the half yearly dividends be paid him; and further, that a part of the corpus of the fund be sold out each half year and paid him, sufficient to make up his total half yearly income to $P\frac{1\frac{1}{2}}{100}$, M_1 being calculated so that the whole shall be exhausted in n years.

$\frac{100\,M_1}{100-\delta}$ is the sum of consols at first. At the first half year $\frac{100\,M_1}{100-\delta}\cdot\frac{1\frac{1}{2}}{100}$ is the dividend payable, and a sum of consols must be sold out to realize $\left\{P\frac{1\frac{1}{2}}{100} - \frac{100\,M_1}{100-\delta}\frac{1\frac{1}{2}}{100}\right\}$ cash. The sum of consols thus sold will be $\frac{100}{100-\delta_{\frac{1}{2}}}\left\{P\frac{1\frac{1}{2}}{100} - \frac{M\,1\frac{1}{2}}{100-\delta}\right\}$, and there remains $\frac{100\,M_1}{100-\delta}\left\{1+\frac{1\frac{1}{2}}{100-\delta_{\frac{1}{2}}}\right\} - P\frac{1\frac{1}{2}}{100-\delta_{\frac{1}{2}}}$, or $\frac{100\,M_1}{100-\delta}(1+r_{\frac{1}{2}}) - P(1+r_{\frac{1}{2}}) + P$. After the end of the second half year there will clearly remain $\left\{\frac{100\,M_1}{100-\delta}(1+r_{\frac{1}{2}}) - Pr_{\frac{1}{2}}\right\}(1+r_1) - Pr_1$; or $\frac{100\,M_1}{100-\delta}(1+r_{\frac{1}{2}})(1+r_1) - P(1+r_{\frac{1}{2}})(1+r_1) + P$. After n years will remain

$$\frac{100\,M_1}{100-\delta}(1+r_{\frac{1}{2}})(1+r_1)\ldots(1+r_n) - P(1+r_{\frac{1}{2}})\ldots(1+r_n) + P.$$

As the fund is to be exhausted in n years, this $=0$. Therefore,

$$M_1 = \frac{100-\delta}{100}P - \frac{100-\delta}{100}\frac{P}{(1+r_{\frac{1}{2}})\ldots(1+r_n)} = M - M_2.$$

A's and B's shares together, as found by this method, are together equal to the present value of the sum of consols. Viewed from either side the result is equally correct.

Suppose the sum to be invested, not in consols, but in some

other security, the rate of interest at which the sum is calculated to be accumulated should be founded on the price of such other securities, instead of consols; for in that case B would be entitled to demand such a sum as would secure his reversion, not on consol security—for that he would not have, if A enjoyed the sum intermediately—but on such security as the sum might now stand invested on. So if the sum be invested on mortgage, the rate of interest derived from such mortgage should be the basis of the calculation.

Mr. Jellicoe's plan, as applied to this case, would consist in calculating A's and B's interests separately, adopting as the basis of calculation a current rate of interest, irrespective of the interest derivable from the securities in question. Calculated by this rule, A's and B's interests together do not equal the total value of the consols, which appears sufficiently to prove that one or both is wrong. The two shares together, according to this rule—if the assumed rate of interest be greater than the interest from consols, or whatever other stock be the one in question—would be less than the present produce of the stock; and one is puzzled to know what to do with the remainder. Mr. Jellicoe attempts to solve the difficulty by dividing it rateably between A and B, according to their calculated shares of the fund. This must be ascribed either, on the one hand, to an arbitrary assumption that, an error being found in the result, the error in A's calculated share is to the error in B's as A's share is to B's share; or, on the other hand, to a still more arbitrary attempt to divide between A and B that to which neither has any claim.

Mr. Jellicoe's reasoning neglects the superior goodness of security on consols over that which may realize 5 per cent. In the same way, it must neglect the inferior goodness of a 6 per cent. security, if it be invested on that instead of consols; and so, if the sum be invested on Turkish 6 per Cents, or the like, the shares of A and B must be calculated as in the case of consols. In the case of consols, Mr. Jellicoe would say that the sum of the present values of A's share and B's taken separately is less than the two sold together, and the surplus has to be divided between them. In the case of Turkish Sixes, it must, according to the same reasoning, be said that the sum of A's and B's shares taken separately is greater* than the two sold presently together, which is absurd.

* On the contrary, the sum in every case must be less, no matter what the security. —Ed. *A. M.*

I have taken the case in which A's interest is for a definite term of years, as the simpler, but the principle must be the same where his interest is for life. If A, aged x years, have a life interest in the stock instead of an interest for n years, then, if d_x, &c. denote the deaths in each year by the tables, and the deaths in the two half years of any one year be supposed equal,

$$M_2 = \frac{100-\delta}{100} \cdot \frac{P}{2l_x}\left\{\frac{d_x}{1+r_{\frac{1}{2}}} + \frac{d_x}{(1+r_{\frac{1}{2}})(1+r_1)} + \frac{d_{x+1}}{(1+r_{\frac{1}{2}})(1+r_1)(1+r_{1\frac{1}{2}})} + \ldots\right\}$$

the series being continued to the latest value of d_w in the tables.

It may be said, that in the case of a life interest a deduction must be made from the calculated value (by whatever means calculated), on account of the *uncertainty* of the possession. This may be true where the sale is to a stranger; but A, in buying B's reversion, is the one person to whom this is no disadvantage, and who therefore cannot fairly claim such deduction; which would also be the case with B if he bought A's life interest. In dealings between these two persons, therefore, the calculated value should be used without deduction on either side.

Historical Sketch of the Life of CHARLES GILL, *Esq.,* late Actuary of the Mutual Life Insurance Company of New York.*

ONLY a few brief months have passed and gone since it was our painful duty to chronicle the death of one who, while in life, occupied a prominent place and acted a conspicuous part amid the bustling scenes of his fellow-citizens. His unexpected death was mourned—and justly so—as a great municipal, if not a national, calamity; while his memory and worth are still cherished with sincere respect by a numerous circle of private friends and public admirers. The question is repeated with as much concern and earnest solicitude now, as when it was first propounded, "Who shall occupy the place left vacant by the removal of Walter R. Jones, late President of the Atlantic Mutual Insurance Company?" And still echo answers, Who? Although varied, numerous, and im-

* As Professor Gill was personally known to many of our readers, and corresponded at one period with this *Journal*, we believe that we shall have general approval in extracting this sketch of his life from the *United States Assurance Gazette*. It will be seen that, independently of his merits as a philosopher and mathematician, he was a very estimable person in every respect.—ED. *A. M.*